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Sergio Barbero
Jacob Rubinstein

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Sergio Barbero
Instituto de Óptica (CSIC)
Serrano 121, 28006 Madrid, Spain
E-mail: sergio.barbero@csic.es

Jacob Rubinstein
Technion, Israel Institute of Technology
Department of Mathematics
Haifa 32000, Israel

Abstract. The need for affordable and sustainable ophthalmic systems for measurement and correction of refraction is well recognized. Power-adjustable spectacles based on the Alvarez principle (transversal lateral movement of two lenses) have emerged as an innovative technology for this purpose. Within this framework, our aim is to design a new power-adjustable sphero-cylindrical refractor. The system is comprised of two lenses and three independent lateral movements. The lenses have a planar and a third-degree polynomial surface. They are arranged with their planar surfaces in contact, so that the incoming light is only refracted by two surfaces. First, we present the theory of such a system. Second, we propose an optical design methodology. Third, we provide a design example capable of measuring sphere powers ranging from −5.00 D to +5.00 D and cross-cylinders from −2.00 D to 2.00 D. Finally, a prototype of the lenses was manufactured using free-form machining. [DOI: 10.1117/1.OE.52.6.063002]

Subject terms: sphero-cylindrical refraction; astigmatic systems; power adjustable lenses; Alvarez lenses.

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1 Introduction

In optometry practice, measuring a person's refraction means to determine the refractive error, i.e., the spherical power and astigmatism discrepancy from emmetropia status of that person. There are two techniques to measure refraction: objective and subjective. The objective refraction is performed with the aid of a retinoscope or an autorefractor. In a subjective test, the person reads a chart through different lenses and his/her vision is evaluated with the help of a visual acuity test. While an objective refraction is carried out to give a first estimate of the actual refraction, eventually the prescription is determined by the subjective refraction, since it provides the spectacles that gives the person his/her preferred best vision.[1]

In practice, the subjective refraction is done with a set of trial lenses. These lenses are located in front of the eye either with the help of a trial lens frame or a phoropter. While this test is the gold standard in optometry, it needs to be carried out by trained personnel making use of expensive and not very portable equipment. Therefore, this test is often not available to many people in developing countries.

As a consequence, several alternatives have been recently proposed to provide an affordable refraction measurement for the developing world. Two of them are the inFOCUS Focometer and the Adspec. The Focometer[2] is a two lens Badal system, where the power change is realized through a longitudinal movement of one lens with respect to the other. The Adspec[3] is a spectacle comprising a flexible membrane covering a fluid that can be pumped into or out of the lens. When the fluid is pumped into the lens, its shape becomes more curved, and increases the power of the lens. Although the Adspec was originally developed for spherical refraction correction, its potential as a refraction measurement tool has been recently analyzed.[4,5] A detailed analysis of the accuracy of these devices as subjective refractors was done by Smith.[6] Other techniques based on the Alvarez principle could also be used (see Ref. 7 and references therein). However, all these techniques are limited because they can only measure the spherical component of refraction. If the eye has astigmatism, a sphero-cylindrical refraction is needed. Our goal is to propose a new affordable technology not only for spherical, but also for sphero-cylindrical refraction.

The standard notation in optometry practice describes a sphero-cylindrical refraction with three quantities: the best vision sphere, the cylinder power (both measured in diopters), and the cylinder axis orientation (expressed in degrees). However, Stokes[8] observed that the astigmatism of the eye can be corrected with the help of a pair of cylindrical lenses of equal magnitude but opposite sign. Therefore, instead of the cylinder power and its axis, one can more conveniently describe cylinder refraction as the linear combination of two cross-cylindrical lenses.[9] Based on Stokes' ideas, an optical device comprising two cylindrical lenses was invented to measure cylindrical subjective refraction. This is the so-called Jackson-cylinder.[10] In the Jackson-cylinder, the cylinder power is varied by rotating one of the lenses with respect to the other, around a common optical axis. The amount of attainable power ranges from zero when the axes of both lenses are aligned, to a maximum of twice the power of one of the lenses when the axes of both lenses form a right angle. So, the Jackson-cylinder can be considered as a variable cylindrical power device.

An important novelty in the concept of a variable power cross-cylinder was introduced by Humphrey.[11] Alvarez had previously found[12] that certain type of lenses, with a surface profile described by $A(x^3/3 + xy^2)$ when laterally

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shifted, can produce a spherical power variation. Humphrey proposed[11,13] a slightly different lens surface profile, $A(x^3/3 - xy^2)$, to generate variable astigmatism. This lens has the special property that when shifted in the x-direction produces a variable cross-cylinder oriented at 0 to 90 deg, and a 45 to 135 deg cross-cylinder is obtained when the lateral movement is performed along the y-direction.[14] A lens with a pure cubic term (x^3) also induces a pure cylinder when moved in the x-direction.[15] Humphrey's ideas were the origin of the Humphrey Vision Analyzer (HVA),[13,16] an innovative tool in subjective refraction at that time.[13]

Humphrey's idea of using lens lateral shifts rather than lens rotations has two advantages. First, instead of controlling the cylinder and the cylinder axis, the Humphrey system controls the two astigmatic power components (cross-cylinder at 0 to 90 deg and at 45 to 135 deg). This is particularly convenient when the cylinder magnitude is small. In such cases, the adjustment of the cylinder axis becomes difficult, because the precision in the axis alignment depends on the cylinder power. Second, in contrast to a mechanical device with rotations, a mechanical system with lateral movements is easier to implement.

The notion of a cross-cylinder is closely related to the concept of dioptric power matrix. This matrix has been shown to be a powerful mathematical tool to describe the sphero-cylindrical refraction data. Extensive literature can be found on this subject (e.g., see Ref. 17 and references therein), but here we interpret it as the amount whereby the optical system changes the incident wavefront curvature[18] in a single refractive surface. Therefore, the dioptric power matrix is computed as the Heissan surface matrix multiplied by the differences in refraction indices between the media at both sides of the refractive surface.

It has been pointed out that the HVA is particularly suitable for the dioptric space representation.[17] In spite of clear technological advantages, the HVA has not experienced widespread use because of its high cost among other reasons. The HVA comprises three pair of lenses (on the whole six lenses): first pair to adjust the sphere, second pair to adjust one cross-cylinder, and finally a third pair for the complementary cross-cylinder.

We now propose a new optical system based on displacements of cubic-type lenses. Yet, our system, instead of being based on three pair of lenses, comprises only two lenses. So we reduce by four the number of optical lenses for a sphero-cylindrical refractor with respect to the HVA. In doing so, the system could be mounted in a low cost, and portable spectacle lens mount that could be easily transported in developing countries.

In the next section, we present the theory (within paraxial optics) of the sphero-cylindrical refractor. In Sec. 3, we explain the optical design methodology used to design such a system. In Sec. 4, we present a prototype of the manufactured lenses. Finally, we provide a discussion of the main results of our work.

2 Optical Theory of the Sphero-Cylindrical Refractor

The system is comprised of two lenses. Each lens has a planar surface and a cubic-type surface. The lenses are arranged with their planar surfaces in contact, so that the incoming light is only refracted by the two surfaces. The nonplanar anterior lens profile is described by the monkey saddle surface:

$$u(x, y) = A\left(\frac{x^3}{3} + xy^2\right), \tag{1}$$

where A is a constant to be selected. The posterior nonplanar lens profile is:

$$v(x, y) = B\left(\frac{x^3}{3} - xy^2\right), \tag{2}$$

where B is a constant. Three lateral movements are used: δ_{ux} denotes a lateral movement of the front lens along the x-direction, and δ_{vx} and δ_{vy} denote lateral movements of the back lens along the x- and y-directions, respectively. A planar incident wavefront is refracted at the surface u, and after propagation inside the lenses, is refracted at the surface v. To understand the performance of the optical system as seen by the user, it is convenient to define the shifted surfaces: $u_\delta := u(x + \delta_{ux}, y)$, $v_\delta := v(x + \delta_{vx}, y + \delta_{vy})$.

Within the framework of paraxial optics, and ignoring the optical effect of the thickness of the lens, the overall dioptric matrix evaluated at the center point of the system ($x_0 = 0$, $y_0 = 0$) is:

$$\tilde{K}(x_0, y_0) = (n-1)\begin{pmatrix} \frac{\partial^2 u_\delta(x_0,y_0)}{\partial x^2} & \frac{\partial^2 u_\delta(x_0,y_0)}{\partial x \partial y} \\ \frac{\partial^2 u_\delta(x_0,y_0)}{\partial x \partial y} & \frac{\partial^2 u_\delta(x_0,y_0)}{\partial y^2} \end{pmatrix} + (1-n)\begin{pmatrix} \frac{\partial^2 v_\delta(x_0,y_0)}{\partial x^2} & \frac{\partial^2 v_\delta(x_0,y_0)}{\partial x \partial y} \\ \frac{\partial^2 v_\delta(x_0,y_0)}{\partial x \partial y} & \frac{\partial^2 v_\delta(x_0,y_0)}{\partial y^2} \end{pmatrix}. \tag{3}$$

Here, n is the refraction index of the lens. From Eq. (3), we obtain:

$$\tilde{K}(x_0, y_0) = 2(n-1)\begin{pmatrix} -A\delta_{ux} + B\delta_{vx} & B\delta_{vy} \\ B\delta_{vy} & -A\delta_{ux} - B\delta_{vx} \end{pmatrix}. \tag{4}$$

It is straightforward to obtain from Eq. (4) the values of the mean sphere (S), and the two orthogonal cross-cylinder components[19] (C_+, C_x) associated with the dioptric matrix

$$S = 2(n-1)A\delta_{ux}, \tag{5}$$

and

$$C_+ = 2(n-1)B\delta_{vx} \tag{6}$$

$$C_x = 2(n-1)B\delta_{vy}. \tag{7}$$

A schematic drawing of the optical system, compared with respect to the HVA analyzer system, is depicted in Fig. 1.

Mathematically speaking, the dioptric matrix is a symmetric 2×2 matrix. Therefore, it is determined by three independent parameters. As seen in Eq. (4), the three independent linear shifts indeed provide the required three parameters. Referring to Fig. 1 in particular, we note that although the shifts δ_{ux} and δ_{vx} follow the same direction, they induce different optical effects because they set different

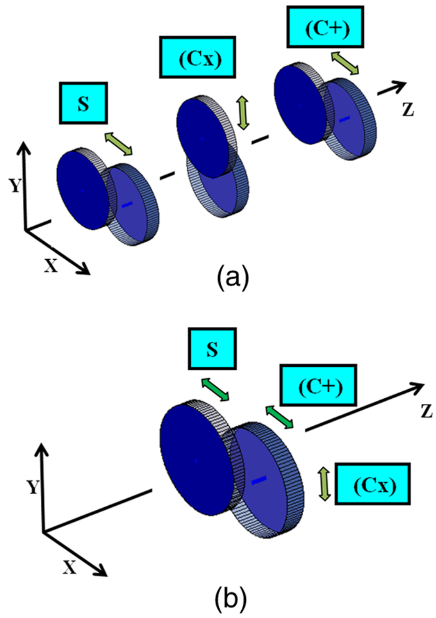

Fig. 1 (a) Humphrey Vision Analyzer (HVA) comprising three pairs of lenses. Each pair controls one of the refractive quantities S, C_x, and C_+. (b) New two lens system. x is the movement of the front lens setting S. x and y are the movements of the rear lens setting C_+ and C_x, respectively.

final configurations of the two-lenses optical system in the user's fixed reference frame.

2.1 *Refraction Procedure*

Equations (5)–(7) show that S, $C_{+,}$ and C_x depend linearly on only one lateral movement each. Therefore, the refraction routine can be performed following the sequence:

1. The best-vision sphere refraction is obtained by means of x-lateral displacement δ_{ux} of the front lens, maintaining the back lens fixed. The sphere, in paraxial approximation, is proportional to the lateral shift according to Eq. (5). In order to attenuate the accommodative response a "fogging technique" can be used (Ref. 1, p. 210), i.e., to start making the eye be effectively myopic, so the initial position is recommended to be −5.00 D.
2. Once the best sphere is fitted, the cross-cylinders are adjusted. First, the best vision cross-cylinder at 0 to 90 deg is obtained by means of x-lateral displacement δ_{vx} of the posterior lens, while keeping fixed the previously moved front lens. In the paraxial approximation, C_+ is proportional to the lateral shift according to Eq. (6). Second, the best vision cross-cylinder at 45 to 135 deg is obtained by means of a y-lateral displacement δ_{vy} of the back lens, which generates a proportional cylindrical-power change [Eq. (7)].

3 Optical Design of a Sphero-Cylindrical Refractor

In previous works, we have shown,[7,20] that the paraxial approximation is not accurate enough to compute sphere power and astigmatism in real-thick optical systems comprising Alvarez-type surfaces. Therefore, we used a more careful procedure[7] to compute these magnitudes across the lenses for different gaze directions. We, thus, trace a pencil of rays around each gaze direction, evaluating the optical-path-difference for each ray at the image plane. These data are used to compute the dioptric matrix. The power and astigmatism are related to the eigenvalues of this matrix. We are now interested in S, C_+, and C_x, which are also directly obtained from the computed dioptric matrix thorugh Eqs. (5)–(7).

Since, in the present work, we are only interested in a refractor set-up, we only need to compute the dioptric matrix for the central viewing direction, although the effects of different gaze directions due to eye rotations or decentering during the refraction measurement will be studied in next subsection.

As an example, we started with a system that provides a sphere power variation from −5.00 to +5.00D, and a cross-cylinder power variation from −2.00 to 2.00 D. This range covers the majority of refractive errors in the general population.[21–23]

We started with a predesign consisting of two lenses, each of which has a planar surface, such that these planar surfaces are in contact. The power variation is achieved by sliding the planar surfaces of the lenses with respect to each other. To facilitate the manufacturing of a first prototype of the lenses, we selected a poly(methyl methacrylate) material ($n = 1.49$). However, a higher refraction index material,[7] such as polycarbonate ($n = 1.586$), would presumably provide better optical results. The central thickness of both lenses are 7.5 and 6 mm, respectively. These high values are needed because of the large diameter of the lenses. The nonplanar surface of the front and back lenses are described by the equations: $u = 1.02 \cdot 10^{-3}(x^3/3 + xy^2)$ and $v = -5.1 \cdot 10^{-4}(x^3/3 - xy^2)$, respectively.The front lens is allowed to be moved a maximum of ±5 mm along the x-direction, and the back lens can be moved up to ±4 mm along both the x- and y-directions. For such a system, we depict in Fig. 2 the power errors for the three sphero-cylindrical quantities (S, C_+, and C_x) for different configurations. The power errors are defined as the differences (in diopters) for different lens shifts, between the nominal paraxial values of S, $C_{+,}$ and C_x according to Eqs. (5)–(7) and the real values computed through the aforementioned procedure.

Figure 2(a) depicts the case where the front lens is moved while keeping the back lens fixed ($\delta_{vx} = 0$ and $\delta_{vy} = 0$). In Fig. 2(b), the front lens is not moved and the back lens is moved only along the x-direction ($\delta_{ux} = 0$ and $\delta_{vy} = 0$). In Fig. 2(c), the front lens is not moved and the back lens is moved only along the y-direction ($\delta_{ux} = 0$ and $\delta_{vx} = 0$). Finally, to evaluate the effect of the combined shifts, Fig. 2(d) shows a situation where the front lens is moved after the back lens was moved by arbitrary distances along the x- and y- directions ($\delta_{ux} = -1.25$ mm and $\delta_{vy} = 3.125$ mm). Figure 2 reveals that the power error associated with each lateral shift [S at Fig. 2(a) and 2(d), C_+ at Fig. 2(b), and C_x at Fig. 2(c)] is kept reasonably low (below 0.20 D), but the other two quantities [C_+ and C_x at Fig. 2(a)–2(d), S and C_x at Fig. 2(b), and S and C_+ at Fig. 2(c)] can have values up to 0.6 D.

Starting from this predesign, we applied an optimization procedure trying to reduce the power errors. The optimization methodology is similar to the one previously proposed by us,[7] yet with some relevant modifications, specifically in the definition of the design parameters, the merit function,

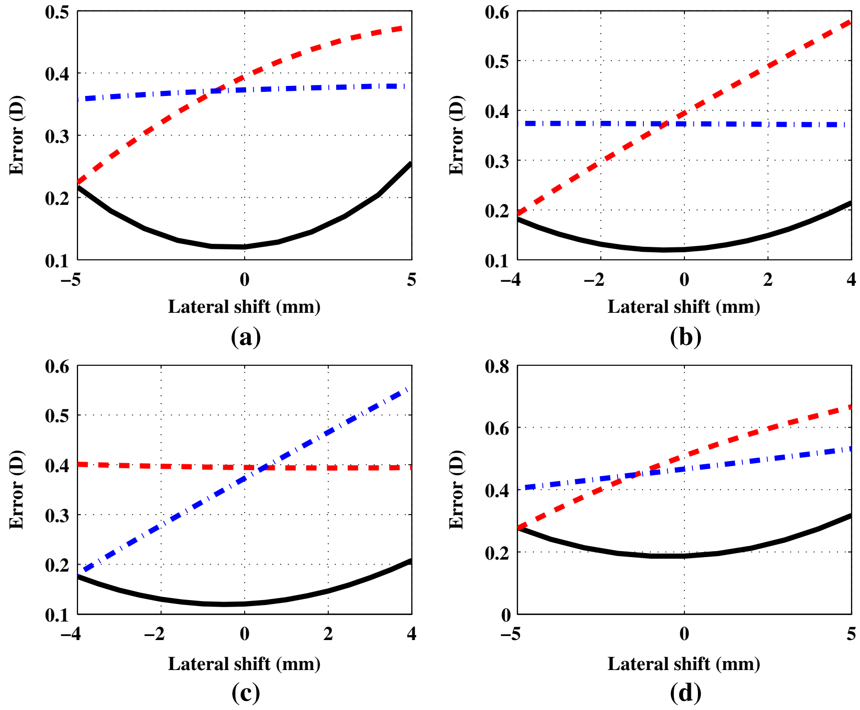

Fig. 2 Sphere (S black solid line), cross-cylinder at 0 to 90 (C_+ red dashed line), and cross-cylinder at 45 to 135 (C_x blue dot-dashed line) power errors (D), for the predesign lens, as function of lateral movement (mm): (a) The abscissa represents the x front lens shift ($\delta_{vx} = 0$ and $\delta_{vy} = 0$). (b) The abscissa represents the x back lens shift ($\delta_{ux} = 0$ and $\delta_{vy} = 0$). (c) The abscissa represents the y back lens shift ($\delta_{vx} = 0$ and $\delta_{vy} = 0$). (d) The abscissa represents the x front lens shift ($\delta_{ux} = -1.25$ mm and $\delta_{vy} = 3.125$ mm).

and the optimization steps. We first defined a general surface taking the form:

$$u(x, y) = \frac{cr^2}{1 + \sqrt{1 - (K + 1)cr^2}} + p_1x^3 + p_2y^3 + p_3xy^2 + p_4yx^2 + p_5x^2 + p_6xy + p_7y^2 + p_8x + p_9y. \tag{8}$$

Here $r = \sqrt{x^2 + y^2}$, and c and K are the radius of curvature and the asphericity of the base conic, respectively. The nonplanar surfaces are described by this equation. The specific optimization problem at hand is symmetric about the $y = 0$ axis. Therefore, in principle one does not need to include terms that have odd powers in y. We do include them for two reasons. One is that sometimes (although not in the present case) minimizers break the problem's symmetry, and, more importantly, we keep an eye toward more sophisticated designs that take into account the eye's movement, which is known to be nonsymmetric about the $y = 0$ axis. For the design, we set the merit function to be:

$$\mathrm{MF} = \frac{\sum_{j=1}^{j=N}(\mathrm{SE}^{1j} + \mathrm{CE}_+^{1j} + \mathrm{CE}_x^{1j}) + \sum_{i=2}^{i=3}\sum_{l=1}^{l=m}(\mathrm{SE}^{il} + \mathrm{CE}_+^{il} + \mathrm{CE}_x^{il})}{N + 6m}. \tag{9}$$

The variables in the equation above are defined as follows:

- The index i denotes three possible configurations. Only the front lens is moved ($i = 1$); only the back lens is moved either along the x-direction ($i = 2$) or the y-direction ($i = 3$).
- The index j denotes a x-lateral shift of the front lens and N is the total number of shifts considered. We set $N = 11$.
- The index l denotes a shift of the back lens either along the x-direction (where $i = 2$) or the y-direction (where $i = 3$). In both cases, m denotes the total number of shifts considered. We set $m = 17$.
- SE^{ij} denotes the power error of S for configuration i and lateral shift j.
- CE_+^{ij} and CE_x^{ij} denote the dioptric error of C_+ and C_x, respectively, for configuration i and lateral shift j.

The merit function was optimized following a cascade approach, where different surface parameters [Eq. (6)] were optimized at two successive steps. In the first step, the parameters c, K, p_1, p_3, p_5, and p_7 of both lenses were optimized. In a second step, all the p_i parameters of both surfaces were optimized. Additionally, an automatic weight adjustment, as described in Ref. 7, was also applied. If the values of SE^{ij}, CE_+^{il}, or CE_x^{il} take values below 0.12 D during the optimization, the algorithm gives zero weights to them.

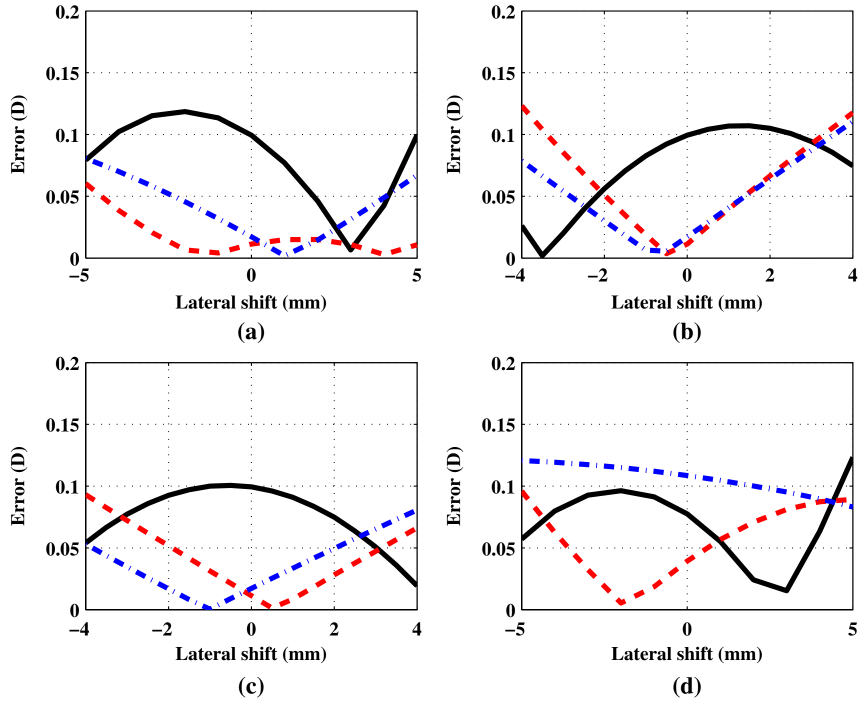

Fig. 3 Sphere (S black solid line), cross-cylinder at 0 to 90 (C_+ red dashed line), and cross-cylinder at 45 to 135 (C_x blue dot-dashed line) power errors (D), for the optimized lens, as function of lateral movement (mm): (a) The abscissa represents the x front lens shift ($\delta_{vx} = 0$ and $\delta_{vy} = 0$). (b) The abscissa represents the x back lens shift ($\delta_{ux} = 0$ and $\delta_{vy} = 0$). (c) The abscissa represents the y back lens shift ($\delta_{vx} = 0$ and $\delta_{vy} = 0$). (d) The abscissa represents the x front lens shift ($\delta_{ux} = -1.25$ mm and $\delta_{vy} = 3.125$ mm).

The optimized surfaces of the form of Eq. (8) had the following coefficients: for the front lens $[c, K, p] = [0.01, 0, 3.5 \cdot 10^{-4}, -3.88 \cdot 10^{-5}, 0.001, -1.52 \cdot 10^{-5}, -8.59 \cdot 10^{-5}, 1.97 \cdot 10^{-4}, 1.44 \cdot 10^{-4}, -0.0332, -7.68 \cdot 10^{-5}]$, and for the back lens: $[c, K, p] = [0.0098, 10^{-4}, 1.68 \cdot 10^{-4}, 4.59 \cdot 10^{-6}, -4.84 \cdot 10^{-4}, -2.22 \cdot 10^{-5}, 2.22 \cdot 10^{-4}, -4.91 \cdot 10^{-4}, -3.36 \cdot 10^{-4}, 0.0159, 1.14 \cdot 10^{-4}]$. We present the performance of the optimized lens in Fig. 3. This figure shows the power errors for the three sphero-cylindrical quantities (S, C_+, and C_x) for different configurations in the same way as in Fig. 2. The optical performance is significantly improved compared with the initial design. In particular, in the optimized design the power errors are mostly below 0.1 D in absolute value.

3.1 *Optical Quality for Off-Axis Gaze Directions*

As mentioned above, although for refraction measurement the viewing direction must be aligned with the optical system, disalignments or eye movements during the measurements are possible. Therefore, it is useful to check how S, C_+, and C_x are modified for different lines of sight around the central viewing direction.

When a person looks at objects located outside the primary line of sight, the eyeball rotates as the line of sight changes. This movement induces an additional rotation, called cyclotorsion, about the new line of sight considered as axis. The cyclotorsion, which is controlled by the extraocular muscles, is not an independent movement; rather, it is uniquely determined by the gaze direction according to a rule proposed by Listing. This law states that the cyclotorsion is the same as "if the eye had been turned around a fixed axis perpendicular to the initial and final directions of the line of fixation" (Ref. 24, vol. 3, p. 48). In other words, the eye rotates about an axis that is in the plane orthogonal to the primary line of sight.

One option to express Listing's law is via Euler angles.[24] However, we present an alternative and more computationally efficient procedure based on vector calculus. Let $\tilde{K}(x, y)$ be the dioptric matrix for an eye in the primary line of sight. In this position, we use a base coordinate system formed by $\vec{x}$, $\vec{y}$, $\vec{z}$. For instance, we use the coordinate vectors to be $\vec{x} = (1, 0, 0)$, $\vec{y} = (0, 1, 0)$, and $\vec{z} = (0, 0, 1)$. Let the eye move to a new line of sight described by the vector $\vec{\zeta} = (r_1, r_2, r_3)$. According to Listing's law, this rotation of the eye is done with respect to an axis in the ($\vec{x}$, $\vec{y}$) plane that we denote by $\vec{a}$. This vector must be, therefore, orthogonal to both $\vec{z}$ and $\vec{\zeta}$, namely

$$\vec{a} = (-r_2, r_1, 0)/\sqrt{r_1^2 + r_2^2}. \tag{10}$$

Obviously $\vec{a}$ is only defined if there is some rotation, i.e., $r_1^2 + r_2^2 \neq 0$. The rotation angle can now be expressed via $\cos\theta = \vec{z} \cdot \vec{\zeta}$.

Given $\vec{x}$ and θ, we can find the rotation of any vector $\vec{v}$ around the axis $\vec{a}$ applying Rodrigues' rotation formula:

$$\vec{v}_{\text{rot}} = \cos\theta\vec{v} + (1 - \cos\theta)(\vec{a} \cdot \vec{v})\vec{a} + \sin\theta\vec{v} \times \vec{a}. \tag{11}$$

In particular, we can apply this vector operation to the original $\vec{x}$ and $\vec{y}$ and find their rotated versions $\vec{\xi}$ and $\vec{\eta}$, respectively. Thus, we derive the coordinate system formed by $\vec{\xi}$, $\vec{\eta}$, $\vec{\zeta}$. The dioptric matrix for different gaze directions is computed through ray tracing and optical-path-difference computations as explained in the previous section,[7] but now using the new coordinate system $\tilde{K}'(\xi, \eta)$.

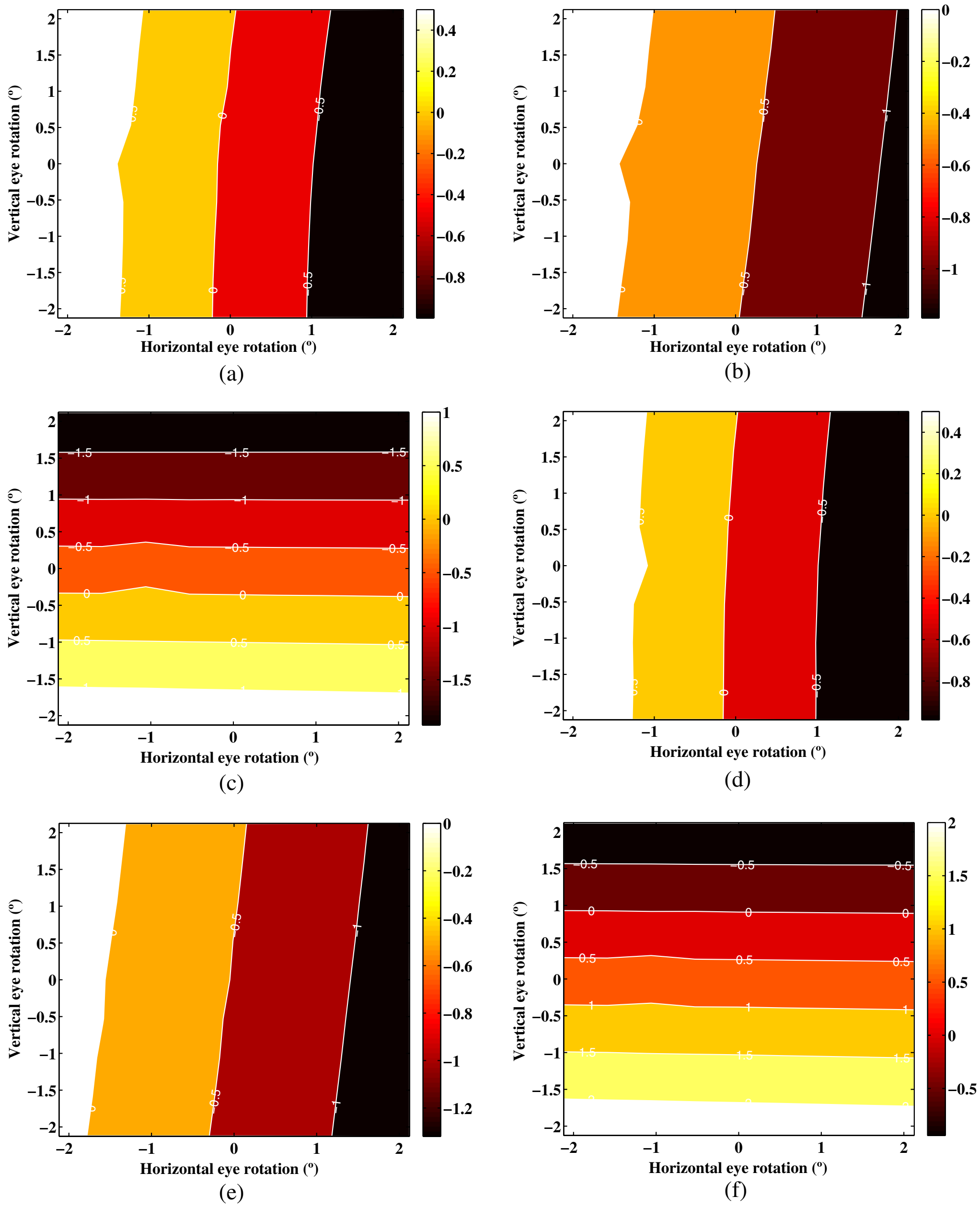

Fig. 4 Power error (D) performance, power deviation from the value at the center of the optimized system for different eye rotations. The $x - y$ axes are the horizontal and vertical eye rotations, respectively. The errors in S (a, d), C_+ (b, e), and C_x (c, f) are given for two different configurations: $\delta_{ux} = -5$ mm, $\delta_{vx} = 0$, and $\delta_{vy} = 0$ (a–c) and $\delta_{ux} = -5$ mm, $\delta_{vx} = 2.5$ mm, and $\delta_{vy} = 2$ mm (d–f).

We computed the power errors in S, C_+, and C_x with respect to the desired values for a region covering 2 deg $\times$2 deg of eye rotations. Figure 4 shows these values for the optimized system. The errors are given for two different configurations. The first configuration, $\delta_{ux} = -5$ mm, $\delta_{vx} = 0$, and $\delta_{vy} = 0$ is shown in Fig. 4(a)–4(c), respectively. The second configuration, $\delta_{ux} = -5$ mm, $\delta_{vx} = 2.5$ mm, and $\delta_{vy} = 2$ mm is shown in Fig. 4(d)–4(f), respectively.

4 Prototype Manufacturing

There are few reports on the manufacturing of Alvarez lenses. A review of some of them is provided in Ref. 25. The most promising application of the technology is arguably for eyeglasses. Indeed, several eyeglasses models are already being manufactured and are available on the market.[26–28] However, there is no available information on the manufacturing methods and accuracy of these models.

Our prototype was manufactured by Light Prescriptions Innovators using ultra-precision diamond turning, specifically a five-axis Freeform Generator (Moore Nanotech® 350 FG).[29] Figure 5(a) and 5(b) shows the computer-aided design (CAD) files used for the manufacturing, and Fig. 5(c) and 5(d) depicts photos of the manufactured lenses. Figure 5(a) and 5(c) shows the front lens, and Fig. 5(b) and 5(d) shows the back lens.

The manufacturing precision has been evaluated with regard to surface finish and lens-form accuracy. While the lens-form accuracy provides a quality measurement of the reliability in machining a specific surface shape, the surface finish describes how well the optical-machined surface has

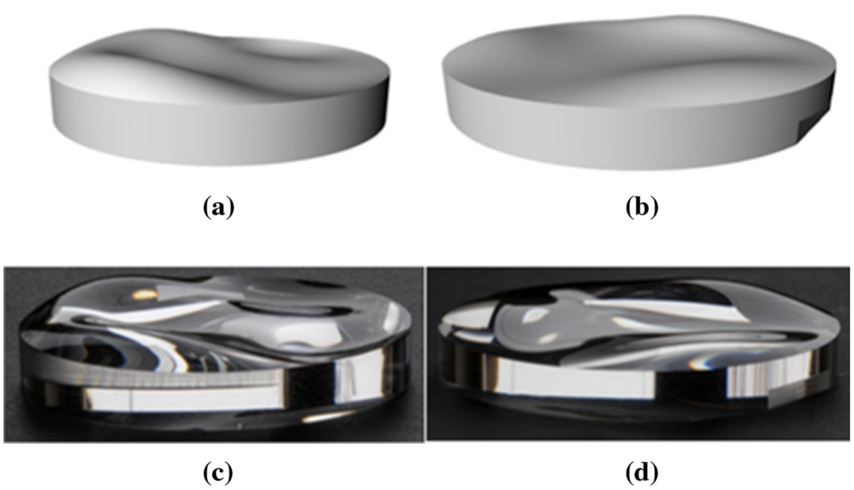

Fig. 5 CAD files used for the manufacturing: (a) front and (b) rear lenses. Photos of manufactured lenses: (c) front and (d) rear lenses.

been polished. Polishing is particularly important for ophthalmic lenses because of its effect on the lens transparency.

The surface roughness was evaluated at four different locations across the lens using a scanning white light interferometer (NewView™ 6300, Zygo®). The average values of root mean square deviation and average roughness (Ra, average deviation from the mean) were 8 and 6.25 nm for the rear lens and 8.25 and 6 nm for the front lens, respectively.

The surface shape was examined using a three-dimensional laser line scanning technology (Surveyor CS-2822, Laser Design Inc., Minneapolis, MN) at the Polytechnic University of Madrid in Spain. The sag coordinate was measured at around 12,800 points on a circular area of 30 mm (lens size).

We compare in Fig. 6 the nominal elevation data [Fig. 6(a) for the front lens and Fig. 6(c) for the rear lens] with respect to the measured values of the manufactured lenses [Fig. 6(b) for the front lens and Fig. 6(d) for the rear lens]. The absolute errors in the elevation data are shown in Fig. 6(e) and 6(f) for the front and the rear lenses, respectively. The mean absolute error is 8.1 and 8.7 μm for the front and the rear lenses, respectively; however the errors at some parts take values up to 30 μm.

5 Discussion

The World Health Organization (WHO) called for a worldwide initiative to provide "sustainable, affordable, equitable, and comprehensive eye care services to prevent avoidable blindness" in developing countries.[30] When considering avoidable blindness due to uncorrected refractive errors, there are at least two plausible approaches to achieve this goal. One is to provide the patients with high quality power-adjustable lenses. We demonstrated a design procedure for such lenses.[7] A second plausible way to achieve the goal above of the WHO is to adequately measure refractive errors, and then supply very cheap but aesthetically appealing single vision lenses. This approach requires portable refractors that can provide reliable measurement of sphero-cylindrical refraction. In this manuscript, we have proposed a design of such a refractor. It is comprised of just two lenses and three linear movements. The mechanism underlying the system has been explained within the paraxial optics framework.

Certain quality standards establish the tolerance in refractive power in single vision lenses. The ANSI Z80.1-2005 (Ref. 31) standard set admissible discrepancies in the power up to 0.13 D in lenses with nominal powers below 6.5 D. For single vision lenses of higher nominal powers, the admissible errors are 2% over the nominal power. These magnitudes are close to recent measurements of minimum values of sphere and cylinder noticeable by the human eye, which are around 0.15 D (Ref. 32). Figures 2 and 3 show that a design procedure, based on optimization of the paraxial predesign, is needed to obtain a lens system that satisfies such tolerances. However, to ensure optical performance of acceptable quality, the available dioptric range is somewhat limited. Thus, our design example is limited to $\pm$5.00 D for the sphere and $\pm$2.00 D for the cross-cylinders.

Our new sphero-cylindrical refraction measurement device, using only two lenses, could be implemented in a spectacles-type frame mount. However, some work must be done to design a mechanical device realizing the three independent movements. We have previously designed a spectacle lens frame that provides two independent lateral movements.[33] In an on-going project, we are designing a system to allow for a third independent movement as well.

Figure 4 shows that power errors increase very rapidly, even for small eye rotations. This implies that the refractor would be very sensitive to misalignments or eye movements during the refractions measurement. Also, these variations imply that optical aberrations, even with perfect alignment, are significant. One way to control these effects is to use a variable size diaphragm to be located in front of the mechanical frame. An additional direction, that we plan to pursue in the future, is to replace the shape function given by Eq. (8) by a free-form surface. Our experience in lens design indicates that this will further improve the optical quality of our lenses, although in the narrow field of vision required for a refractor, the improvement is not expected to be dramatic.

Figure 6 depicts the errors in the manufacturing process. The errors take high values at some parts of the lenses (30 μm). Deviations within a few micrometers are usual in free-form machining.[25] However, as reported elsewhere,[25] the surface figure measurement errors are probably larger than that of the machining. This effect is probably amplified in our lenses, where the total depth variation is unusually

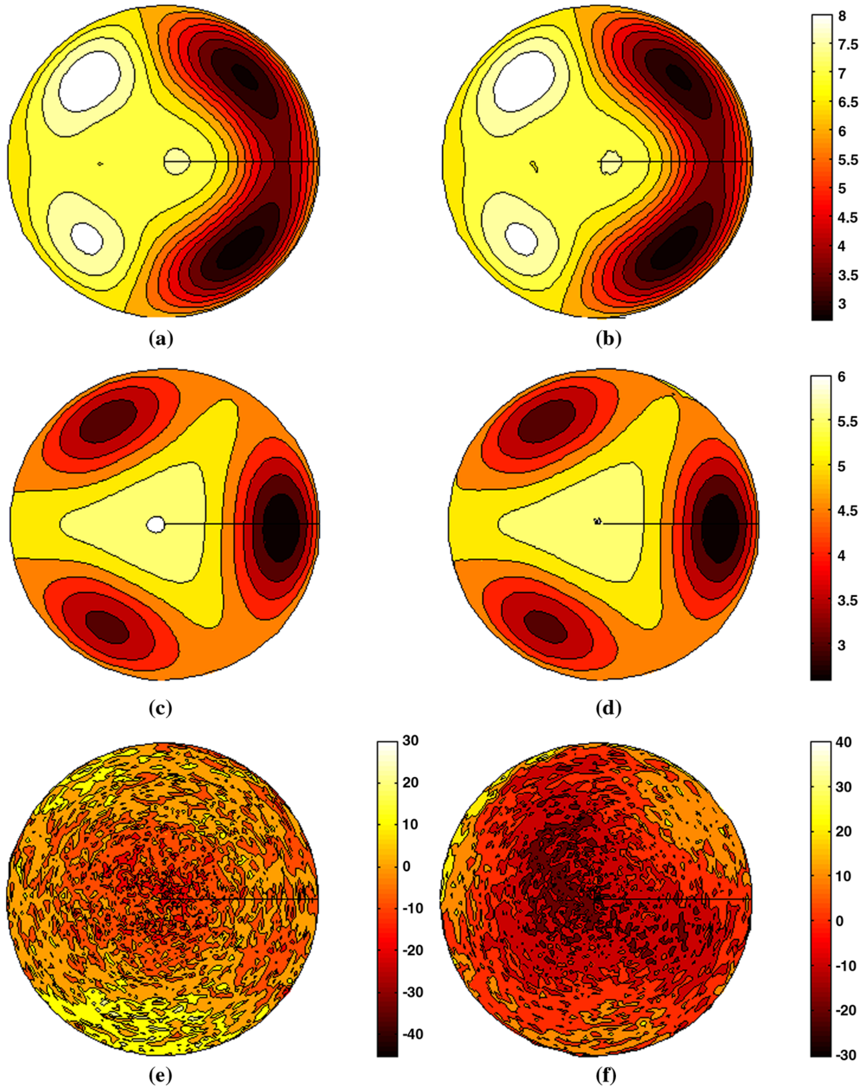

Fig. 6 Nominal elevation maps (mm): (a) front and (c) rear lenses. Measured elevation maps (mm): (b) front and (d) rear lenses. Absolute error maps (μm): (e) front and (f) rear lenses. The black solid line shows the positive x-axis.

high: 5.75 and 3.42 mm for the front and rear lenses, respectively. Nevertheless, it has been suggested that the tolerance errors for a single ophthalmic surface must not exceed 0.06 D, which corresponds approximately to 80 μm of error in the lens form at the center of the lens.[34] As a consequence, we can be confident that the lens-form accuracy achieved here is enough for our purposes.

Finally, we note that our device could also serve to measure the amplitude of accommodation, as a replacement of the so-called "minus lens technique" (Ref. 1, p. 230). Briefly, in this technique, the subject, with his/her far vision refraction corrected, is asked to look to a fixed target (typically located at a distance of 40 cm). Then, minus lenses, in 0.25 D steps, are sequentially located in front of the subject until the eye is not able to correct this myopic stimulus by means of the accommodative response. The amplitude of accommodation is the highest amount of minus lenses introduced. Analogously, this minus power stimulus can be introduced easily with our device by only moving the lens which controls the sphere.

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Sergio Barbero obtained his BS degree in physics (optics) from Universidad de Zaragoza (1999), and his PhD degree in visual optics in 2004 in the Instituto de Optica, CSIC, Madrid. He was awarded, from 2004 to 2006, with a Fulbright postdoctoral fellowship to perform research at Indiana University in the field of wavefront sensing and reconstruction. In 2009, he joined the faculty of the Instituto de Optica (CSIC). His science interests lie in optical design, visual optics, and optical system modeling.

Jacob Rubinstein obtained his BSc and MSc in applied mathematics from Tel Aviv University and Technion, respectively, and his PhD in mathematics from New York University. In 1988, he joined the Technion Mathematics Department. He is also an adjunct professor at Indiana University. He is involved in ophthalmic lens design since 1990, and he holds a number of international patents for his inventions. He is a fellow of the American Association for the Advancement of Science.