1040-5488/11/8811-1391/0 VOL. 88, NO. 11, PP. 1391–1392
OPTOMETRY AND VISION SCIENCE

CORRESPONDENCE

Thomas Young's Investigations in Gradient-Index Optics

TO THE EDITOR

It is a common place in the history of science that problems and their solutions obtained in the past fall into oblivion but are later rediscovered in the future. It appears to be the case with one of Thomas Young's optical problems, recently reviewed by Atchison and Charman.[1,2]

The problem is that of perfect imaging of parallel rays with a spherical surface, which appears in Young's 1801 article: *The Bakerian Lecture. On the mechanism of the eye*[3] (Proposition VI, page 32).

Young claimed to find "the law by which the refraction at a spherical surface must vary, so as to collect parallel rays to a perfect focus."[3] The law, as written in the 1801 article is as follows: $m = \sqrt{m^2 \pm 2nv}$, where *m*/*n* is "the ratio of the sine of the angle of incidence to the sine of the angle of refraction."[3]

In a corollary, Young suggested that a double convex lens with such property could serve to conjugate two equidistant points.

In this letter, I show the equivalence of Young's problem to the so-called Tarkhanov lens,[4] which has emerged with the modern research on GRIN optics.

First, I review Atchison and Charman[1] comments on Young's result.

Following geometrical considerations, Atchison and Charman[1] found a different equation:

$$m = \sqrt{m_o^{\ 2} - 2nv(m_o - n)} \qquad (1)$$

Atchison and Charman clarified Young's equation by introducing the variable m_o as the refractive index at the vertex of the surface. But still the equations are different. My hypothesis is that the absence of the term $(m_o - n)$ in Young's equation is just a typographical error, because it is at the end of the square root. However, the presence of the $\pm$ symbol is still obscure, because it suggests that m could have two possible solutions: $m = \sqrt{m_o^{\ 2} + 2nv(m_o - n)}$ or $m = \sqrt{m_o^{\ 2} - 2nv(m_o - n)}$. Anyhow, the most reasonable thing is to believe that Thomas Young knew the correct solution (Eq. 1) to his problem.

Besides, Atchison and Charman assume that what Young had in mind, when he stated his problem, was a medium with an axial refractive index distribution. However, if Young just thought about an ideal lens where the rays are somewhat refracted at the surface, following his law, and once inside the lens the rays simply follow straight lines, it is straightforward to prove that the law would still be the same.

It is well known that Cartesian ovals, and not spheres, are the refracting surfaces, separating two homogeneous mediums, that perfectly focus incoming parallel rays. In a 1988 article,[5] Vladimir Ivanovich Tarkhanov established a close related problem to eliminate spherical aberration for rays coming from a point located at infinity with an axial refractive index medium, bounded by a rotationally symmetric surface. As a special case, when this surface is a sphere, the problem is reduced to Young's problem. For this case, and when setting that the homogeneous refractive index out of the surface is the unity, Tarkhanov obtained—using a geometrical procedure and also applying Fermat's principle—the following equation:

$$m(z) = \frac{(f + r)}{\sqrt{2z(f + r) + f^2}} \qquad (2)$$

where m(z) is the variable refractive index at the surface, f is the focal length, z is the sagitta (Fig. 1), and r is the radius of curvature of the surface.

The same equation was independently derived by Dueck et al.,[6] though using a normalized radius of curvature.

A planoconvex lens with an axial refractive index distribution given by Eq. 2 was named "Tarhanov's lens."[4] I thank Roman Ilinsky for clarifying that "Tarkhanov" and "Tarhanov" is the same person: the difference is due to different variants of translating Russian names into the Latin alphabet.

To show the equivalence between Young's law and the Tarkhanov lens, I apply Fermat's principle to Young's problem

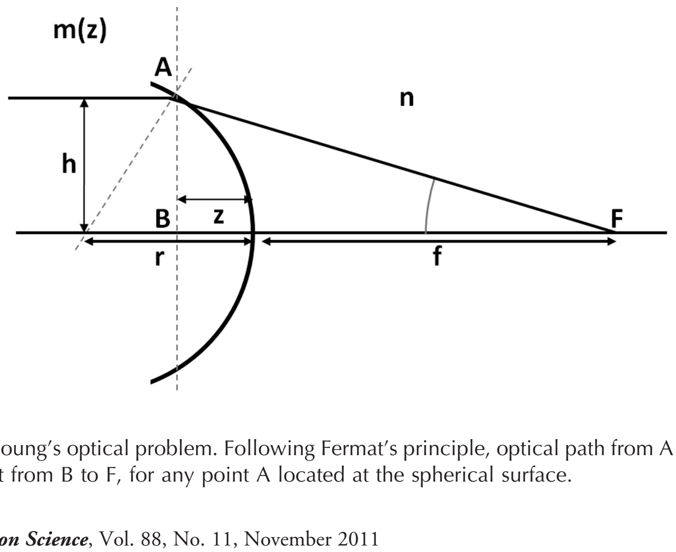

FIGURE 1.
Scheme of Young's optical problem. Following Fermat's principle, optical path from A to F must be equal to that from B to F, for any point A located at the spherical surface.

as done by Tarkhanov,[5] but with an arbitrary homogeneous refractive index n.

The geometry of the problem is shown in Fig. 1. A ray moving parallel to the optical axis and inside the axial refractive index medium intersects the spherical surface at point A. Afterward, the refracted ray intersects the focal point F. B is a point in the optical axis with the same axial coordinate as A. For perfect focusing, the optical path from A to F must be equal to that from B to F for any point A at the surface.

This implies:

$$n\sqrt{(f+z)^2+h^2} = nf + \int_0^z m(z)dz \quad (3)$$

From here, it is possible to obtain m(z) by differentiation. Later, using the sagitta formula of a circle and the versed sine relation ($z = vr$), we get:

$$m(v) = \frac{n(r+f)}{\sqrt{vr(2r-vr)+(f+vr)^2}} \quad (4)$$

It is clear that Eq. 4 is equivalent to Eq. 3 in Atchison and Charman's article.[1] Henceforth, it is possible to follow Atchison and Charman procedure[1] to obtain Eq 1.

It is also clear that Eq. 2 (Tarkhanov lens) is a specific case of Eq. 4. when n = 1.

The same way as Young noticed that "the same law will serve for a double convex lens, in the case of equidistant conjugate foci,"[3] Tarkhanov wrote: "by combining two such systems, one can obtain an anaberrational system for a point on the axis at a finite distance."[5]

Tarkhanov also realized[7] that the same axial distribution of the refractive index of a perfectly focusing lens is valid for a perfectly diverging lens.

More recently, other types of lenses have been proposed to provide lenses free of spherical aberration with spherical surfaces. Ilinsky proposed a gradient-index meniscus lens with a spherical refractive index distribution.[4]

ACKNOWLEDGMENT

I thank Roman Ilinsky for bringing references 4 and 5 to my attention.

Sergio Barbero, PhD
Instituto de Óptica "Daza de Valdés"
Consejo Superior de Investigaciones Científicas
Serrano 121
28006 Madrid, Spain
e-mail: sergio.barbero@csic.es

REFERENCES

1. Atchison DA, Charman WN. Thomas Young's investigations in gradient-index optics. Optom Vis Sci 2011;88:580–4.
2. Atchison DA, Charman WN. Thomas Young's contributions to geometrical optics. Clin Exp Optom 2011;94:333–40.
3. Young T. The Bakerian lecture. On the mechanism of the eye. Phil Trans Royal Soc Lond 1801;91:23–88.
4. Ilinsky R. Gradient-index meniscus lens free of spherical aberration. J Optics (A) 2000;2:449–51.
5. Tarkhanov VI. Calculation of axial distribution of refractive-index, leading to formation of an anaberrational system. Soviet J Optical Technol 1988;55:91–4.
6. Dueck RH, Vaughn JL, Hunter BV. Optical design with inhomogeneous glass: the future is here. In: Fischer RE, Johnson RB, Juergens RC, Smith WJ, Yoder PR, Jr., eds. Lens Design, Illumination, and Optomechanical Modeling: SPIE Proceedings, Vol. 3130. Bellingham, WA: SPIE;1997:32–40.
7. Tarkhanov VI. Design of an aplanatic lens with axial refractive-index gradient. Soviet J Optical Technol 1990;57:295–7.

AUTHORS' RESPONSE

We appreciate the interest that Dr. Barbero has taken in this topic and for drawing attention to relevant gradient index work of which we were not aware. We are pleased to have verification of our Eq. 2.

It is indeed possible that our understanding of Young's intention is not correct.

We agree wholeheartedly with Dr. Barbero's opening sentence about the loss of knowledge of early work. The interval between Young and Tarkhanov's work was nearly 200 years, and it is a little sad that Young's notoriously difficult presentation of his ideas made it so hard for later authors to benefit from some of his profound insights.

It is interesting that the utility of index gradients was first recognized in the context of eyes, not only with Maxwell and Gullstrand but also with Exner in studies of the ommatidia of compound eyes. It took a long time for lens designers to become interested in the possibilities offered by gradient index!

David A. Atchison, MScOptom, PhD, DSc
Visual and Ophthalmic Optics Laboratory
School of Optometry and Institute of Health & Biomedical Innovation
Queensland University of Technology
Kelvin Grove, Queensland, Australia
e-mail: d.atchison@qut.edu.au
W. Neil Charman, DSc
Dept. of Optometry & Vision Sciences
University of Manchester
Manchester, United Kingdom